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## GRATING ORIENTATIONAL NONLINEARITY: UTILIZATION IN OPTICAL FEEDBACK ARRANGEMENTS

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Abstract Operation of passive ring mirror and double-conjugation scheme utilizing orientational nonlinearity of nematic liquid crystal is investigated theoretically and experimentally.

#### INTRODUCTION

It is well known (see, e.g., 1) that high values of orientational grating nonlinearity of nematics (GRON) enables one to manage nonlinear optical processing of low-power beams, such as millisecond pulses of solid state lasers 2 and CW argon-ion lasers radiation, including optical phase conjugation 2 and self-conjugation3. Nevertheless there is some problem as far as self-conjugation via stimulated backscattering is considered 3. The matter is that for high enough quality of such conjugation one needs interaction length (i.e. sample thickness) exceeding the Fresnel length of pump radiation at least by a factor of 5. Taking into account typical sample thickness of about 100 µm, this requirement is practically never satisfied, thus leading to a poor quality of conjugation 3.

There is however a way of avoiding such difficulties. It lies in utilization of various optical feedback arrangements, the simplest representaive of those being so called passive ring mirror <sup>4,5</sup>. In such case the requirement mentioned above for pump Fresnel length is a great deal softened: the latter must not exceed only the external feedback circuit length (several cm at least), and one can

obtain, in principle, high enough quality of conjugation.

The possibilities for realization of such arrangements utilizing GRON are considered in this paper.

#### GEOMETRY OF INTERACTION

Let's consider a typical geometry of interaction for feedback arrangement mentioned above (Fig.1).

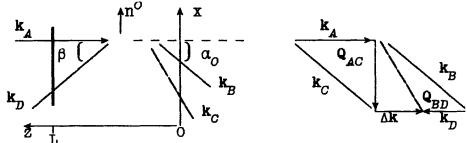


FIGURE 1. Geometry of interaction

Two pump beams A and B which we consider as nondepleted, penetrate a nematic sample nearly counterwards to each other. Each wave undergoes stimulated orientational forward scattering  $(SS)^2$ , the waves C and D being originated, polarizations of those being orthogonal to A and B respectively due to the SS properties<sup>2</sup>.

The case of interest is when scattering of wave A into C and B into D utilize the same (or nearly the same, see further) orientational grating  $\mathbf{Q}_{AC} \sim \mathbf{Q}_{BD}$ , so that the shortened Helmholtz equations for C and D become coupled. In case of birefringent media this requires a certain combination of waves' polarizations (see Fig.1b). Namely if A is of e-type then D is of e-type also, while D and C are both o-polarized.

In such case of two scatterings utilizing the same grating an absolute instability of C and D magnitudes can take place after exceeding some threshold pump intensity, which was broadly discussed for photorefractive crystals (see, e.g.,  $^{4-8}$ ). The case of GRON however differs a great deal from photorefractives due to difference in real and

imaginary parts of effective susceptibilities for those two mechanisms.

If the waves A and B are originated independently, the arrangement in Fig.1 is called double-side conjugate mirror. On the other hand, one can introduce optical feedback, utilizing wave A, transmitted through the sample, to oreate B. Then the waves C and D are also coupled by the same feedback circuit (its amplitude transmittance being  $\zeta$ ) and such arrangement is usually referred to as passive ring mirror.

#### BASIC EQUATIONS

The system of equations for description of four wave mixing mentioned above consists of two shortened Helmholtz equations for slow amplitudes of C and D - and two variational equations  $^1$  for complex amplitudes of the gratings:

$$\frac{\partial \mathbf{C}/\partial \mathbf{z} = -i \left( \mathbf{\theta}_{AC} + \mathbf{\theta}_{BD} \exp \left( i \Delta \mathbf{K} \mathbf{r} \right) \right) \mathbf{A}}{\partial \mathbf{D}/\partial \mathbf{z} = i \left( \mathbf{\theta}_{AC} \exp \left( -i \Delta \mathbf{K} \mathbf{r} \right) + \mathbf{\theta}_{BD} \right) \mathbf{B}}$$

$$\frac{\partial \mathbf{\theta}_{AC}}{\partial \mathbf{t} + \mathbf{\theta}_{AC}} \frac{\partial \mathbf{r}_{AC}}{\partial \mathbf{r}_{AC}} = \mathbf{x} \mathbf{A}^{*} \mathbf{C}$$

$$\frac{\partial \mathbf{\theta}_{BD}}{\partial \mathbf{t} + \mathbf{\theta}_{BD}} \frac{\partial \mathbf{r}_{AC}}{\partial \mathbf{r}_{BD}} = \mathbf{x} \mathbf{B}^{*} \mathbf{D}$$

$$(1)$$

Here amplitudes of the proper waves are calibrated as outside the medium,  $\theta_i$  are proportional to the complex magnitudes of the proper angular deviation of the director  $\delta_c$ :  $\theta_i = 2\pi\epsilon_a \delta_{oi}/\lambda (n_i n_i)^{1/2}$ ;  $\alpha = \epsilon_a^2/16\pi\lambda n_i n_i - \text{effective}$  constant of transient GRON (see<sup>2</sup>).  $\epsilon_a$  stands for dielectric tensor anisotropy,  $\eta$ - orientational viscosity,  $\lambda$ -wavelength of the radiation. As for  $\tau_i$ , those are relaxation times of proper gratings,  $\tau_i = \eta/KQ_i^2$ . Here K is Frank-constant (the uniconstantional approximation is used),  $Q_i$  is the proper grating wave number (see Fig.1). At last,  $\Delta K = Q_{BD} - Q_{AC} = k_A + k_D - k_B - k_C$  is the two gratings' wave mismatching.

From (1) it can readily be seen that for true coupling one needs  $\Delta K_x \equiv 0$ , otherwise two terms in the right

side will produce two different waves outside the medium.

As to boundary and initial conditions, in case of passive ring mirror, which is our experimental point of interest, they are following:

$$C(L,t)=aA(t);$$
  $D(O,t)=\zeta C(O,t);$   $\theta_{\epsilon}(z,O)=\theta_{O,\epsilon}(z)$  (2)

a being some surfacial scattering source. In case of double-side conjugate mirror it is more convinient to take

$$C(L,t)=0; D(0,t)=0; \theta_{t}(z,0)=\theta_{ot}(z)$$
 (3)

The solution of (1,2) or (1,3) depends sufficiently upon pump pulse duration  $\tau_p$ . Let's consider two opposite cases: i)  $\tau_p \ll \tau_{AC,BD}$ ; ii)  $\tau_p \to \infty$  (CW radiation)

#### TRANSIENT RESPONSE

So we let  $\tau_p \ll \tau_{AC,BD}$  and relaxation terms in the left side of material equations (1) can be omitted. We also let  $\theta_{oi}(z)=0$  in order not to overcomplicate the formulae. Moreover, we consider only the case of untilted backscattering wave D:  $\beta=0$  (see Fig.1), i.e.  $\Delta K=0$ , which correspond to experimental observation. Then introducing  $s=D(z,t)/\Delta(t)$  and taking temporal Laplace image of Eqs. (1,2) for passive ring mirror,— one can easily obtain following expression for Laplace image of s:

$$\sum_{g(p)=-\infty}^{\infty} (a\zeta/p) \cdot \frac{\exp(-i\Re(1-R)z/p) - 2/(1+R)}{\exp(-i\Re(1-R)L/p) - 2/R(1+R)}$$
(4)

After some more or less complicated mathematical transformations final expression for the reflectivity of such transient passive ring mirror will be following:

$$R^{NL} = |s(L, v)|^{2} = a^{2}R|1 - 2 \frac{1+R}{1-R} \sum_{i=1}^{\infty} (\frac{2R}{1+R})^{n} I_{o}(2(in(1-R)v)^{1/2}|^{2})$$

$$v = 2L \int_{0}^{1} |A(t')|^{2} dt'$$
(5)

Here I denotes Bessel function of imaginary argument. Results of numerical calculations for  $R^{NL}/a^2(v)$  are presented in Fig.2a) for various values of feedback constant

R. It should be noted that despite of rather complicated form, expression (5) reveals quite obvious physical meaning: each term in the right side is a result of n-fold transient SS in some cavity, losses in which are determined by R.

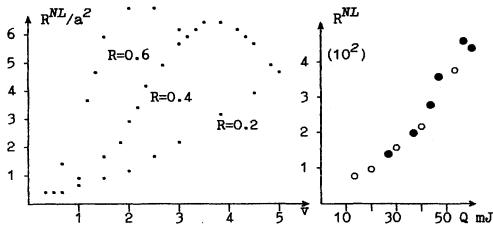


FIGURE 2 Reflectivity of transient passive ring mirror.

The results above were verified experimentally for the case of 1-ms duration free running ruby laser pulse (τ,≈3.5ms). Passive ring mirror was arranged using 70 μm thickness planarly aligned 50B sample. Beams A and B, intersecting at  $\alpha_{n} \approx 13^{\circ}$ , were focused, forming focal waists within the sample of ≈70µm in diameter. Experimental results for RNL vs current value of pump exposition Q (see (5)) is presented in Fig.2b). For R≈0.2, which was the experimental value, experimental results coinside well enough with numerical calculations. Rather high (≥0.5) quality of conjugation (i.e. ratio of the conjugated component power and the total retroreflectred power) was obtained when the angular divergence of the pump beams was spoiled by a factor of 5 using an aberrator, - so that the pump Fresnel length became twice shorter than the feedback circuit length.

#### CW PUMP RESPONSE

In this case it is more convenient to take a=0,  $Q_{oi}\neq 0$ , which in fact is the real case. Then system (1,2) or (1,3) allows exponential temporal solution:  $\theta_{i}$ , C,D ~  $\exp(\mu t)$ ,  $\mu$  maybe having imaginary part corresponding to Stokes frequency shift of C,D with respect to A,B. For the sake of simplicity we allow  $t_{AC}=t_{BD}=t_{o}$  which requires  $|\Delta K| \ll |Q_{i}|$ . This condition is really satisfied unless  $\beta$  is too large. Substituting C=Uexp( $\Delta Kz$ )/ $\zeta$ ,  $\xi=z/L$  into (1,2,3) we obtain following system, quite analogous to  $^{7}$ :

$$\frac{dD/d\xi = \iota \Gamma(U + RD)}{dU/d\xi = -\iota \Gamma(U + RD) + \iota QU} \Gamma = \frac{\Gamma_o/(1 + \nu_1 + \iota \nu)}{(6)}$$

Here Q is wave mismatching Q=AKL;  $\Gamma_o = 2 \tau_o L |A|^2$  is the increment for SS  $A \rightarrow C$ ;  $\nu_i = \tau_o Re \mu$ ,  $\nu = \tau_o Im \mu$ . Boundary conditions for passive ring mirror and double-side conjugate mirror are (7) and (8) correspondently:

$$U(1)=0;$$
  $D(0)=U(0)$  (7)

$$U(1)=0;$$
  $D(0)=0$  (8)

Both problems (6,7) and (6,8) are typical problems of parametric instability, and instability thresholds are found from proper secular equations with additional requirement  $v_1=0$ . Those equations for ring mirror and double-conjugator are following:

$$\exp(\lambda_2 - \lambda_1) = \frac{RQ - \Gamma R(1 - R) - \epsilon R(\lambda_2 - \lambda_1) - \epsilon \lambda_2}{RQ - \Gamma R(1 - R) - \epsilon R(\lambda_2 - \lambda_1) - \epsilon \lambda_2}$$
(9)

$$\exp(\lambda_2 - \lambda_1) = \frac{\Gamma R + \epsilon \lambda_1}{\Gamma R + \epsilon \lambda_1} \tag{10}$$

$$\lambda_{1,2} = -0.5i\{Q + (1-R)\Gamma \mp (\Gamma^2(1-R)^2 + 2\Gamma(1+R)Q + Q^2)^{1/2}\}$$

These equations allow analytical solution only in two ca-

ses: i)Q=0, that mean untilted signal waves - and ii)  $Im\Gamma=0 \Rightarrow \nu=0$ , i.e. unshifted signal waves' frequencies. For Q=0 we immediately obtain (11) and (12) for ring mirror and double-conjugator respectively.

$$v_n = \ln((1+R)/2R)/2\pi n; \Gamma_{on} = 2\pi n \cdot \frac{1+\ln^2((1+R)/2R)/4\pi^2 n^2}{1-R};$$
 (11)

$$v_n = \ln R/2\pi n$$
;  $\Gamma_{on} = 2\pi n \cdot \frac{1 + \ln^2 R/4\pi^2 n^2}{1 - R}$ ;  $n = 1, 2...$  (12)

For  $\nu$ =0 solution for double-conjugator is absent, while for passive ring mirror:

$$Q_n = -\pi \ln \left\{ \frac{(1-R)(1+R)^2}{4+12R+9R^2-3R^3} \right\}^{1/2}; \quad \Gamma_{on} = -\pi \ln \left\{ \frac{(1-R)^{-1}(1+2R)^2}{4+12R+9R^2-3R^3} \right\}^{1/2} (13)$$

double-conjugator considered was in detail elsewhere 7,8, we'll examine only passive ring mirror. It is obvious that in real experimental situation one will obtain **D** wave with Q and  $\nu$  corresponding to the lowest threshold value  $\Gamma_o$ . Lowest branches of (11,13) for reasonable value R=0.2 give  $\Gamma_O \approx 8$  for Q=0 and  $\Gamma_O \approx 1$  for  $\nu=0$ . The latter is no doubt preferable, but then Q≈-3, which requires  $\alpha_{o} \approx \beta \approx 0.1$  rad for typical L,  $\epsilon_{ii,1}$ . Such high values of intersection and tilting angles are very hard to realize experimentally using a strongly focused CW radiation by several reasons. Among those are problems of overlapping pump beams within the sample, failure of the feedback circuit by large values of  $\beta$  etc.

For neither Q nor  $\nu$  being zero only numerical solution of (9) is possible. Results of such solution are presented in Fig.3.

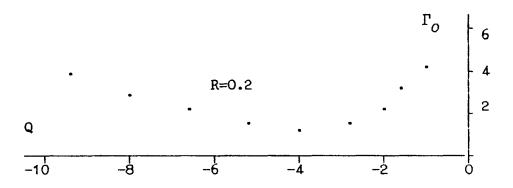


FIGURE 3 Dependense of  $\Gamma_o$  upon Q.

It can be seen, that the optimal pair of (Q,v) lies somewhere nearby (-3,0) being thus practically not observable

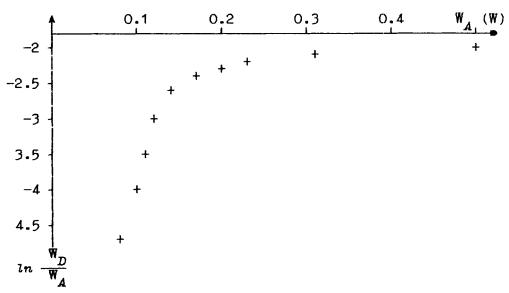


FIGURE 4

experimentally.

Yet an attempt was made to observe the ring mirror signal from 100  $\mu$ m-thick planar 5CB sample – using counterpropagating pump waves **A** and **B** of o- and e-polarizations correspondently (CW argon ion laser radiation) focused into  $\approx 20\mu$ m-diameter spots within the sample. Fig. 4 shows the experimental dependence of backscattered e-wave power  $W_D$  upon pump one  $W_A$ .

It can readily be seen that the curve is a typical

SS-type one, achieving saturation at  $W_A$  values corresponding to  $\Gamma_{O}$  =4. Although the (Q=0) threshold  $\Gamma_{O}$  was exceeded, we didn't observe any threshold like anomalies at such values of W, perhaps due to strong pump depletion via SS.

So it is preferable perhaps for CW pump case to utilize double-side conjugator, which do not require feedback circuit, although having twice higher optimal threshold 7  $\Gamma_o \approx 2$ .

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